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Linear Optical Protocol for Generation of Greenberger-Horne-Zeilinger State within a Network

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Abstract We propose a linear optical protocol to generate Greenberger-Horne-Zeilinger (GHZ) state under control of a number of controllers in terms of optical elements. The proposed setup involves simple linear optical elements, $(N-1)$-photon polarization entangled state, and conventional photon detectors. We show that the protocol can be successfully realized with a certain probability.

Keywords Linear optical elements · GHZ state · Conventional photon detectors

Entanglement, in particular the entanglement between distant particles, is not only a key ingredient for the tests of quantum nonlocality [1], but also an important physical resource in achieving tasks of quantum computation and quantum communication [2]. Hence, generation of entangled states and its further applications are immensely important. The power of entanglement is exploited in a number of quantum communication protocols, such as quantum teleportation [3, 4], quantum secret sharing [5], quantum cryptography [6], quantum secure direct communication [7–12], quantum cloning machine [13], and so on. These concepts motivated an intensive research in the generation and the manipulation of entangled states.

For tripartite systems, the W class states and the Greenberger-Horne-Zeilinger (GHZ) class states are two different important classes of genuine tripartite entanglement. The first class, the W state, is represented by the state $|W\rangle = \frac{1}{\sqrt{3}}(|001\rangle + |010\rangle + |100\rangle)$. This state shows perfect correlations, but the violation is weaker than for the second class GHZ state, which is represented by the state $|GHZ\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$. This state is usually referred to as "maximally entangled" in several senses, e.g., it violates Bell inequalities maximally.

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The two class states can't be converted to each other by local operations and classical communications (LOCC) and show different behaviors if one qubit is traced out. As two classes of important quantum resources, great effort has been taken to studying of entangled state generation in the past years [14–27]. Among them the cavity-quantum-electrodynamics (CQED) systems are always paid more attention. This is due to the fact that cold and localized atoms are not only the source of local entanglement, but also well suited for storing quantum information in long-lived internal states. For example, In Refs. [14–17], the authors have proposed protocols to generate of multi-particle entangled state in cavity. On the other hand, due to photons are the natural source for fast and reliable transport of quantum information over long distances, so many works have been proposed to generate photons entangled states using optics elements [21–29]. For example, Zou et al. [25] have proposed a protocol to generate GHZ state of four separate qubits using linear optical elements. Duan et al. [26] used the analogous approach to prepare entangled states of N atoms. Very recently, Su et al. [27] have presented the first experimental results generating continuous variable quadripartite GHZ entangled state of electromagnetic fields. The approach based on indistinguishability was shown to have a lot of advantages, among them the robustness is the most distinct one.

Controlled teleportation theory, first published by Ref. [30], is to make the quantum information be regenerated by a receiver via the control of agents in a network. Only when all agents collaborate, the receiver can restore the original quantum state. If any agent does not cooperate, the original quantum information cannot be fully recovered. In this point of view, the controlled teleportation is safer than its ordinary counterpart. Motivated by this protocol [30], we propose an alternative protocol to prepare GHZ state with linear optical elements and conventional photon detectors. The realization of this protocol is appealing due to the fact that quantum state of light is robust against the decoherence and photons are ideal carriers for transmitting quantum information over long distances.

In this paper, we consider generation of the GHZ state within a network consisting of an arbitrary $N \geq 3$ remote parties named $P_1, P_2, \ldots, P_N$: a known polarization photon GHZ state needs to be prepared between any two parties under control of all the remaining parties. For convenience, in the revised version we will quote some descriptions and notations in the original protocol [31, 32].

Without loss of generality suppose that P_1, P_N, and $P_2, P_3, \ldots, P_{N-1}$, are the generation parties and controllers, respectively. Before performing the network generation protocol, the parties $P_1, P_2, \ldots, P_{N-1}$ should share an $(N-1)$-photon entangled polarization state of the form

$$|\Phi\rangle_{12\ldots N-1} = \frac{1}{\sqrt{2}}(|H\rangle^{\otimes N-1} + |V\rangle^{\otimes N-1})_{12\ldots N-1}, \tag{1}$$

which photons $1, 2, \ldots, N-1$ are in possession of $P_1, P_2, \ldots, P_{N-1}$, respectively. P_N have two photons, a photon in the horizontal polarization state $|H\rangle$ and another in the vertical polarization state $|V\rangle$.

If P_N and P_k [$(k \in \{1, 2, \ldots, N-1\})$ one of the $N-1$ parties P_{N-1}] want to generate GHZ state, they need the help of other parties P_m ($m \in \{1, 2, \ldots, N-1\}$ and $m \neq k$). Without the help of the controllers P_m, the controlled generation protocol failed, even through, only one of the controllers not agree to help P_N and P_k, the protocol failed too. In this paper, we suppose that all the controllers P_m agree to help P_N and P_k (for example P_1) to generate the GHZ state, they take some operations on their photon shown in Fig. 1.

Firstly, the controller (P_2) first sends his/her photon 2 pass through a quarter-wave plates (QWP), whose action is given by transformation $|H\rangle \rightarrow \frac{1}{\sqrt{2}}(|H\rangle + |V\rangle)$ and $|V\rangle \rightarrow$

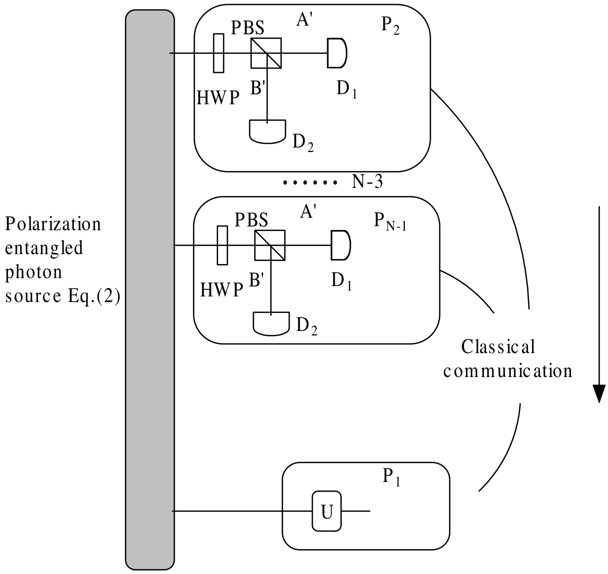

Fig. 1 Schematic diagram for controlled preparation of GHZ state within a network. $D_{1(2)}$ is conventional photon detector, PBS is polarizing beam splitter, HWP is half-wave plate, U is the $\pi/2$-phase shift P

$\frac{1}{\sqrt{2}}(|H\rangle - |V\rangle)$. After QWP, the state of the channel (1) changes into

$$|\Phi\rangle_{12\ldots N-1} = \frac{1}{2}\{|H\rangle_2(|H\rangle^{\otimes N-2} + |V\rangle^{\otimes N-2})_{134\ldots N-1} + |V\rangle_2(|H\rangle^{\otimes N-2} - |V\rangle^{\otimes N-2})_{134\ldots N-1}\}. \tag{2}$$

Then he/she sends the photon 2 pass through a PBS, which transmits horizontal $|H\rangle$ polarization and reflects vertical $|V\rangle$ ones. At the outputs of PBS we obtain

$$|\Phi\rangle_{12\ldots N-1} = \frac{1}{2}\{|H\rangle_2^{A'}(|H\rangle^{\otimes N-2} + |V\rangle^{\otimes N-2})_{134\ldots N-1} + |V\rangle_2^{B'}(|H\rangle^{\otimes N-2} - |V\rangle^{\otimes N-2})_{134\ldots N-1}\}, \tag{3}$$

where A' and B' are output paths of PBS, as is shown in Fig. 1. In order to realize controlled generation, P_2 performs a measurement with the conventional photon detectors (see Fig. 1). P_2 will inform P_1 of his/her measurement result C via a classical communication. If his/her measurement result is $|H\rangle$, $C = 0$; if the measurement result is $|V\rangle$, $C = 1$. The communication costs is 1 cbit since there are two possible results. After that, All the other controllers repeat this process for the photons $3, 4, \ldots, N-1$, respectively, and inform P_1 of their measurement results C_n ($n = 3, 4, \ldots, N-1$) via classical communication. The total communication costs are $N-2$ cbits.

P_1 carries out the following calculation:

$$C_{23\ldots N-1} = C_2 \oplus C_3 \cdots \oplus C_{N-1}, \tag{4}$$

where the $\oplus$ denotes an addition mod 2. If $C_{23\ldots N-1} = 0$, the channel in (1) will be transformed into

$$|\Phi\rangle_1 = |H\rangle_1 + |V\rangle_1; \tag{5}$$

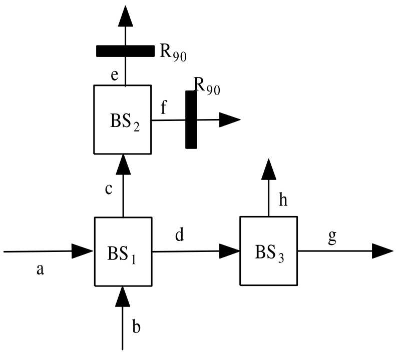

Fig. 2 Schematic diagram for generating of GHZ states with linear optical elements. BS_1, BS_2 and BS_3 denote 50:50 beam splitters and R_{90} denotes a quarter-wave plate

if $C_{23\dots N-1}=1$, the channel in (1) will be will be transformed into

$$|\Phi\rangle_1 = |H\rangle_1 - |V\rangle_1, \tag{6}$$

which can be transformed into (5) by applying a $\pi/2$-phase shift P to change the sign of the polarization state $|V\rangle$.

After the above operations, the state of photon 1 in (7). After normalization, the state in (7) will be

$$|\Phi\rangle_1 = \frac{1}{\sqrt{2}}(|H\rangle_1 + |V\rangle_1). \tag{7}$$

The principle of our scheme for generating a three-photon GHZ state is shown in Fig. 2. P_1 injects his/her photon into the first 50:50 beam splitter (BS_1) from the mode a. P_N sends his/her photons into the BS_1 from the mode b. The whole state of the three photons is

$$|\Psi\rangle_1 = \frac{1}{\sqrt{2}}(|H\rangle + |V\rangle)_a |HV\rangle_b, \tag{8}$$

in which the numbers a and b in the subscript are the spatial modes a and b of BS_1, respectively. The modes a and b are mixed at the BS_1, whose reflectivity and transmissivity are independent of polarizations. The BS_1 with the state transformation

$$a^+_{a,j} \to \frac{\sqrt{2}}{2}(a^+_{c,j} - a^+_{d,j}), \tag{9}$$

$$a^+_{b,j} \to \frac{\sqrt{2}}{2}(a^+_{c,j} + a^+_{d,j}), \tag{10}$$

where $j = H, V$ and $a^+_{a,j}$ and $a^+_{b,j}$ denote the creation operations of the output and input modes a and b.

After passing the BS_1, the whole state of the three photons evolves to be

$$\begin{aligned}|\Psi\rangle_1 &= \frac{1}{\sqrt{2}}(|H\rangle + |V\rangle)_a |HV\rangle_b \\ &\to \frac{\sqrt{2}}{4}(|2_H 1_V\rangle_c |0\rangle_d + |2_V 1_H\rangle_c |0\rangle_d + |0\rangle_c |2_H 1_V\rangle_d + |0\rangle_c |2_V 1_H\rangle_d \\ &\quad + |2_H\rangle_c |1_V\rangle_d + |2_V\rangle_c |1_H\rangle_d + |1_H\rangle_c |2_V\rangle_d + |1_V\rangle_c |2_H\rangle_d),\end{aligned} \tag{11}$$

where $|2_H 1_V\rangle_c$ stands for the state of two horizontally polarized photon and one vertically polarized photon of mode c; $|0\rangle_d$ means that there is neither a vertically polarized nor a horizontally polarized photon of mode d. From (9) we know that there are four possible cases after BS_1: (1) all photons in mode c, (2) all photons in mode d, (3) a photon in mode c and two photons in mode d, (4) two photons in mode c and a photon in mode d.

We only consider those terms that contain two photons in the mode c and one photon in the mode d or vice versa to generate a three-photon GHZ state. Clearly, only these terms contributes to the successful operation. Therefore, if we postselect the successful events, after the BS, (11) changes into

$$|\Psi\rangle_3 = \frac{1}{2}(|2_H\rangle_c|1_V\rangle_d + |2_V\rangle_c|1_H\rangle_d + |1_H\rangle_c|2_V\rangle_d + |1_V\rangle_c|2_H\rangle_d), \tag{12}$$

with the probability $P_{pro_1} = 50\%$. After the BS_2 and the BS_3, (BS_2 and BS_3 are the same as that of BS_1 in this paper) and we consider only those terms that the three photons come out of the two BSs from three different modes, we obtain a useful output state as follows:

$$\begin{aligned}|\Psi\rangle_4 = \frac{\sqrt{2}}{8}(&|H\rangle_e|H\rangle_f|V\rangle_g + |V\rangle_e|V\rangle_f|H\rangle_g + |H\rangle_e|H\rangle_f|V\rangle_h + |V\rangle_e|V\rangle_f|H\rangle_h \\ &+ |H\rangle_e|V\rangle_g|V\rangle_h + |H\rangle_e|V\rangle_g|V\rangle_h + |H\rangle_f|V\rangle_g|V\rangle_h + |V\rangle_f|H\rangle_g|H\rangle_h),\end{aligned} \tag{13}$$

with the probability $P_{pro_2} = 50\%$.

With a quarter-wave plate R_{90} which is used to complete the transformation $|H\rangle \leftrightarrow |V\rangle$ after the mode e and another after the mode g, the output state becomes the standard GHZ state

$$\begin{aligned}|\Psi\rangle_5 = \frac{1}{4}\Big[&\frac{1}{\sqrt{2}}(|H\rangle_e|H\rangle_f|H\rangle_g + |V\rangle_e|V\rangle_f|V\rangle_g) \\ &+ \frac{1}{\sqrt{2}}(|H\rangle_e|H\rangle_f|H\rangle_h + |V\rangle_e|V\rangle_f|V\rangle_h) \\ &+ \frac{1}{\sqrt{2}}(|H\rangle_e|H\rangle_g|H\rangle_h + |V\rangle_e|V\rangle_g|V\rangle_h) \\ &+ \frac{1}{\sqrt{2}}(|H\rangle_f|H\rangle_g|H\rangle_h + |V\rangle_f|V\rangle_g|V\rangle_h)\Big],\end{aligned} \tag{14}$$

the total success probability of generating a three-photon GHZ state in $|\Psi\rangle_5$ is $50\% \times 50\% = 25\%$.

In summary, we have proposed a protocol to prepare photon GHZ state within a network, based on multiphoton interference, ancillary entangled photon states, and conventional photon detectors only to distinguish the vacuum and nonvacuum Fock number state. Let us briefly discuss the characters of our protocol. Firstly, the protocol presented here requires the conventional photon detectors only to distinguish the vacuum and nonvacuum Fock number state. A sophisticated single-photon detector distinguishing one or two photon state is unnecessary. Secondly, the setup is absolutely passive linear optics without the use of fast polarization modulator. Thirdly, if and only if all the controllers agree to cooperate with P_1 and P_N, the controlled protocol can be realized. Without the help of the controllers, P_1 and P_N can not generate the photon GHZ state, so the protocol failed. In this point of view, the present protocol is safer than its ordinary counterpart.

In an experimental scenario, our encoding circuit based on postselection strategy has been demonstrated experimentally in Refs. [33, 34] and recently developed quantum dot techniques might eventually provide a triggered source of entangled photon pairs [35, 36]. Experimental techniques for single-photon detection have made tremendous progress [37–39]. What we used also consists of linear optical elements and photon detectors, which have been widely used to entangle photons. In particular, the similar optical setups have been used to successfully prepare W (GHZ) states of photons in experiment [40, 41]. But, experimental difficulty with the present protocol should be point out. The protocol is based on the interference of photons from modes a and b. This requires that they must arrive simultaneously at the beamsplitter to an accuracy of a fraction of the coherent length. But this requirement is also met in multiphoton experiments [41–43], and has been solved by locking the pathlength. The second challenge is that our protocol requires the deterministic entangled photon states as input source. From Refs. [44, 45] we know that an encoding circuit based on postselection strategy has been demonstrated experimentally [33, 34], where parametric down-conversion was used to generate an entangled pair of photons. But the pairs are created at random times, so that such a source cannot be used in the present scheme. Recently developed quantum dot techniques [35, 36] might eventually provide a triggered source of entangled photon pairs. Thirdly, in terms of practical applicability, our demonstration of the protocol still has some limitations, for example, the imperfection of optical elements and entangled state sources. We hope that with the development of technology in experiment it may be possible to implement the protocol with ease.

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References

1. Bell, S.: Physics **1**, 195 (1964)
2. Bennett, C.H., Divincenzo, D.P.: Nature **404**, 247 (2000)
3. Bennett, C.H., Brassard, G., Crepeau, C., Jozsa, R., Peres, A., Wootters, W.K.: Phys. Rev. Lett. **70**, 1895 (1993)
4. Gordon, G., Rigolin, G.: Phys. Rev. A **73**, 042309 (2006)
5. Zhang, Z.J., Man, Z.X.: Phys. Rev. A **72**, 022303 (2005)
6. Ekert, A.: Phys. Rev. Lett. **67**, 661 (1991)
7. Zhu, A.D., Xia, Y., Fan, Q.B., Zhang, S.: Phys. Rev. A **73**, 022338 (2006)
8. Xia, Y., Song, H.S.: Phys. Lett. A **364**, 117 (2007)
9. Deng, F.G., Long, G.L.: Phys. Rev. A **70**, 012311 (2004)
10. Long, G.L., Liu, X.S.: Phys. Rev. A **65**, 032302 (2002)
11. Deng, F.G., Long, G.L.: Phys. Rev. A **68**, 042315 (2003)
12. Ji, X., Zhang, S.: Chin. Phys. **15**, 1418 (2006)
13. Gisin, N., Massar, S.: Phys. Rev. Lett. **79**, 2153 (1998)
14. Cirac, J.I., Zoller, P.: Phys. Rev. A **50**, R2799 (1994)
15. Gerry, C.C.: Phys. Rev. A **53**, 2857 (1996)
16. Zheng, S.B.: Phys. Rev. Lett. **87**, 230404 (2001)
17. Lange, W., Kimble, H.J.: Phys. Rev. A **61**, 063817 (2000)
18. Hagley, E., Maitre, X., Nogues, G., Wunderlich, C., Brune, M., Raimond, J.M., Hroche, S.: Phys. Rev. Lett. **79**, 1 (1997)
19. Rauschenbeutel, A., Nogues, G., Osnaghi, S., Bertet, P., Brune, M., Raimond, J., Haroche, S.: Science **288**, 2024 (2000)
20. Song, J., Xia, Y., Song, H.S., Guo, J.L., Nie, J.: Europhys. Lett. **80**, 60001 (2007)
21. Feng, X.L., Zhang, Z.M., Li, X.D., Li, S.Q., Gong, S.Q., Xu, Z.Z.: Phys. Rev. Lett. **90**, 217902 (2003)
22. Cabrillo, C., Cirac, J.I., Garcia-Fernandez, P., Zoller, P.: Phys. Rev. A **59**, 1025 (1999)

23. Bose, S., Knight, P.L., Plenio, M.B., Vedral, V.: Phys. Rev. Lett. **83**, 5158 (1999)
24. Simon, C., Irvine, W.T.M.: Phys. Rev. Lett. **91**, 11045 (2003)
25. Zou, X., Pahlke, K., Mathis, W.: Phys. Rev. A **68**, 024302 (2003)
26. Duan, L.M., Kimble, H.J.: Phys. Rev. Lett. **90**, 253601 (2003)
27. Su, X., Tan, A., Jia, X., Zhang, J., Xie, C., Peng, K.: Phys. Rev. Lett. **98**, 070502 (2007)
28. Kim, H., Cheong, Y.W., Lee, H.W.: Phys. Rev. A **70**, 012309 (2004)
29. Zou, X.B., Li, K., Guo, G.C.: Phys. Rev. A **74**, 044305 (2006)
30. Karlsson, A., Bourennane, M.: Phys. Rev. A **58**, 4394 (1999)
31. Tashima, T., Özdemir, S.K., Yamamoto, T., Koashi, M., Imoto, N.: Phys. Rev. A **77**, 030302 (2008)
32. Sheng, Y.B., Deng, F.G., Zhou, H.Y.: Chin. Phys. Lett. **25**, 3558 (2008)
33. Pittman, T.B., Jacobs, B.C., Franson, J.S.: Phys. Rev. A **64**, 062311 (2001)
34. Pittman, T.B., Jacobs, B.C., Franson, J.S.: Phys. Rev. A **69**, 042306 (2004)
35. Benson, O., Santori, C., Pelton, M., Yamamoto, Y.: Phys. Rev. Lett. **84**, 2513 (2000)
36. Akopian, N., Lindner, N.H., Poem, E., Berlatzky, Y., Avron, J., Gershoni, D., Gerardot, B.D., Petroff, P.M.: Phys. Rev. Lett. **96**, 130501 (2006)
37. Kim, J., Takeuchi, S., Yamamoto, Y., Hogue, H.H.: Appl. Phys. Lett. **74**, 902 (1999)
38. Imamoglu, A.: Phys. Rev. Lett. **89**, 163602 (2002)
39. James, D.F.V., Kwiat, P.G.: Phys. Rev. Lett. **89**, 183601 (2002)
40. Eibl, M., Kiesel, N., Bourennane, M., Kurtsiefer, C., Weinfurter, H.: Phys. Rev. Lett. **92**, 077901 (2004)
41. Pan, J.W., Bouwmeester, D., Daniell, M., Weinfurter, H., Zeilinger, A.: Nature **403**, 515 (2000)
42. Bouwmeester, D., Pan, J.W., Daniell, M., Weinfurter, H., Zeilinger, A.: Phys. Rev. Lett. **82**, 1345 (1999)
43. Pan, J.W., Daniell, M., Gasparoni, S., Weihs, G., Zeilinger, A.: Phys. Rev. Lett. **86**, 4435 (2001)
44. Zou, X.B., Zhang, S.L., Li, K.L., Guo, G.C.: Phys. Rev. A **75**, 034302 (2007)
45. Zou, X.B., Li, K., Guo, G.C.: Phys. Rev. A **74**, 044305 (2006)